Cognitive Simulation Model of the Regional Education System

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Professions that are always needed

Physician

teacher

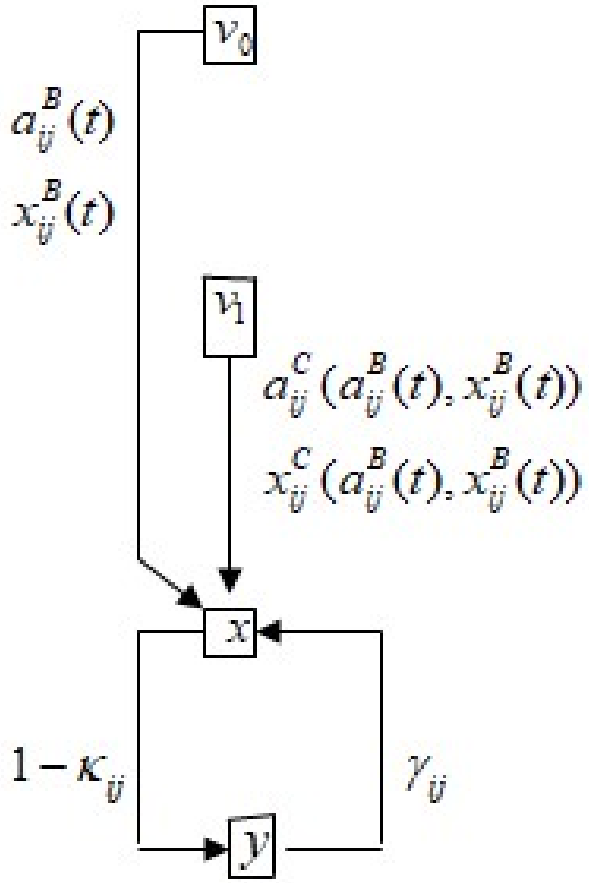
technic

builder

agrarian

Cognitive Map

- N - number of educational organizations in region;
- M - number of specialties;
- T - plan period;
- $x_{ij}(t)$ - number of budget and commercial vacations of j-th direction at i-th organization in t year;
- $x_{ij}^{B}(t)$ - number of budget vacations of j-th direction at i-th organization in t year;
- $x_{ij}^{C}(t)$ - number of commercial vacations of j-th direction at i-th organization in t year;
- $a_{ij}^{B}(t)$ – budget subsidy for one student of j-th direction at i-th organization in t year;
- $a_{ij}^{C}(t)$ – the cost of commercial education of j-th direction at i-th organization in t year;
- γ_{ij}- the coefficient of the influence of employment prospects on the desire for admission at j-th direction at i-th organization;
- $y_{ij}(t)$ - number of employed graduates of j-th direction at i-th organization in t year
- κ_{ij}- a share of unemployed graduates of j-th direction at i-th organization;



Mathematical model

$$J_0 = \sum_{t=1}^{T}\sum_{i=1}^{N}\sum_{j=1}^{M} a_{0j} x_{ij}(t) \to \max$$

$$\sum_{i=1}^{N}\sum_{j=1}^{M} a_{ij}^{B} x_{ij}^{B}(t) \le B$$

$$x_{ij}^{B}(t) \ge 0, a_{ij}^{B}(t) \ge 0$$

$$\sum_{i=1}^{N} x_{ij}(t) \ge x_j^{\min}$$

$$J_i = \sum_{t=1}^{T}\sum_{j=1}^{M}\left[a_{ij}^{B} x_{ij}^{B}(t) + a_{ij}^{C} x_{ij}^{C}(t) - c_i\left(\sum_{j=1}^{M} x_{ij}(t)\right)^2\right] \to \max$$

$$x_{ij}^{C}(t) \ge 0, a_{ij}^{C}(t) \ge 0$$

N - number of educational organizations in region;
M - number of specialties;
T - plan period;
$x_{ij}(t)$ - number of budget and commercial vacations of j-th direction at i-th organization in t year;
$x_{ij}^{B}(t)$ - number of budget vacations of j-th direction at i-th organization in t year;
$x_{ij}^{C}(t)$ - number of commercial vacations of j-th direction at i-th organization in t year;
x_j^{min}- threshold for j-th direction;
a_{0j}- social significance of graduate at j-th direction;
$a_{ij}^{B}(t)$ – budget subsidy for one student of j-th direction at i-th organization in t year;
$a_{ij}^{C}(t)$ – the cost of commercial education of j-th direction at i-th organization in t year;
γ_{ij}- the coefficient of the influence of employment prospects on the desire for admission at j-th direction at i-th organization;
$y_{ij}(t)$ - number of employed graduates of j-th direction at i-th organization in t year
κ_{ij}- a share of unemployed graduates of j-th direction at i-th organization;

$$x_{ij}(t+1) = x_{ij}^{B}(t) + \min\{x_{ij}^{C}(t), (\gamma_{ij} - a_{ij}^{C}(t) y_{ij}(t)\},\ x_{ij}(0) = x_{ij0}$$

$$y_{ij}(t+1) = (1-\kappa_{ij}) y_{ij}(t),\ y_{ij}(0) = y_{ij0},\ i = 1,\dots,N,\ j = 1,\dots,M,\ t = 1,\dots,T$$

Game Reglament

- *Simplifying assumptions*:

  - budget vacancies are filled completely;

  - there is no student's expulsion.

- *Numeric identification:*

  a_{0i}, γ_i, κ_{ij}, c_{ij},x_{ij0}, y_{ij0},$i = 1, \ldots, N, j = 1, \ldots, M.$

- *Control scenarios:*

  for MSE (Centre)

$$\{a_{ij}^B(t), x_{ij}^B(t)\},\ i = 1, \ldots, N, j = 1, \ldots, M, t = 0, \ldots, T-1$$

  for UNIV

$$\{a_{ij}^C(t), x_{ij}^C(t)\},\ i = 1, \ldots, N, j = 1, \ldots, M, t = 0, \ldots, T-1$$

Game Reglament

- Dynamic hierarchical two-level game of *(n+1)* person is considered.
- Stackelberg equilibrium as the solution of Γ1 game is found.
- *At first the Center moves, allocating budget* ***vacancies*** *to the universities in the education orientation at any given time, and allocating certain* ***funding*** *for each budget vacancy.*
- *At the second stage, universities determine the* ***number of commercial vacancies*** *in each education orientation and the* ***cost*** *of education.*

Assumptions for Rostov Region

3 big universities:

DSTU, RSMU, SFU.

6 educational orientation:

Physician, Teacher, Engineers, Builders, Economists, Agrarian

Forecast for 3 years

Specialization of universities in the areas of education

	RSMU	DSTU	SFU
Physician	+		
Teacher		+	+
Engineer		+	+
Builder		+	+
Economist		+	+
Agrarian		+	

Identification

- a_{0i} – significance (importance) of the direction of education for society and the state
- γ_i – economic employment prospects (average salary of specialists)
- κ_{ij} – the share of graduates who are not employed at the specialty
- c_{ij} – the cost of education of i-th university students of the j-th specialty (approximately 80% of the cost of education)
- x_{ij0} – the number of graduates of the j-th specialty of the i-th university in the initial year
- y_{ij0} – the number of graduates of the j-th specialty of the i-th university who got a job in the initial year

Preliminary conclusion

The applicant thinks rationally and correlates the cost of training with possible future income in the procession (salary):

- education for a doctor and an agrarian is relatively expensive, and the average salary is small, cost recovery requires a long period, which reduces the attractiveness of these areas
- average salaries in technical and construction areas are high, this allows someone to quickly recoup costs for commercial education and to increase the attractiveness of these areas
- the costs of obtaining economic and pedagogical education are comparable to the level of average salaries, payback is achieved in the foreseeable future, but rather slowly

If there are no budget vacancies

- $J_i = \sum_{t=1}^{T} \sum_{j=1}^{M} \left[a_{ij}^{C} x_{ij}^{C}(t) - c_i \left(\sum_{j=1}^{M} x_{ij}(t) \right)^2 \right] \rightarrow max$
- $x_{ij}^{C}(t) \geq 0, a_{ij}^{C}(t) \geq 0$
- $x_{ij}(t+1) = min\{ x_{ij}^{C}(t), (\gamma_{ij} - a_{ij}^{C}(t) y_{ij}(t)\},\ x_{ij}(0) = x_{ij0}$
- $y_{ij}(t+1) = (1 - \kappa_{ij}) y_{ij}(t),\ y_{ij}(0) = y_{ij0},\ i = 1, \ldots, N,\ j = 1, \ldots, M,\ t = 1, \ldots, T$

Computations if there are no budget vacancies

	RSMU		DSTU		SFU	
	Vacations	Extra charge	Vacations	Extra charge	Vacations	Extra charge
Physician	(0, 0, 0)	0				
Teacher			(100, 38, 95)	3%	(250, 250, 250)	11%
Engineer			(56, 63, 5)	30%	(86, 98, 9)	18%
Builder			(67, 65, 8)	19%	(28, 27, 5)	1%
Economist			(130, 85, 35)	9%	(250, 250, 250)	8%
Agrarian			(0, 0, 0)	0		

Conclusion if there are no budget vacancies

Over time, the demand for commercial vacancies decreases. In this situation, according to the law of the market, universities are forced to reduce the cost of commercial vacancy.

In SFU, admission to pedagogical and economic orientation is limited by the university's capabilities, does not exceed 250 vacancies.

If there are some budget vacancies

- $J_0 = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{0j} x_{ij}^{B}(t) + \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{0j}\, min\{ x_{ij}^{C}(t), (\gamma_{ij} - a_{ij}^{C}(t) y_{ij}(t)\} \rightarrow max;$
- $x_{ij}^{B}(t) \geq 0, a_{ij}^{B}(t) \geq 0$;
- $\sum_{i=1}^{N} \sum_{j=1}^{M} a_{ij}^{B} x_{ij}^{B}(t) \leq B$
- $\sum_{i=1}^{N} x_{ij}^{B}(t) + \sum_{i=1}^{N} min\{ x_{ij}^{C}(t), (\gamma_{ij} - a_{ij}^{C}(t) y_{ij}(t)\} \geq x_{j}^{min};$
- $J_i = \sum_{t=1}^{T} \sum_{j=1}^{M} \left[a_{ij}^{B} x_{ij}^{B}(t) + a_{ij}^{C} x_{ij}^{C}(t) - c_i \left(\sum_{j=1}^{M} x_{ij}(t) \right)^2 \right] \rightarrow max$
- $x_{ij}^{C}(t) \geq 0, a_{ij}^{C}(t) \geq 0;$
- $x_{ij}(t+1) = x_{ij}^{B}(t) + min\{ x_{ij}^{C}(t), (\gamma_{ij} - a_{ij}^{C}(t) y_{ij}(t)\},\ x_{ij}(0) = x_{ij0};$
- $y_{ij}(t+1) = (1 - \kappa_{ij}) y_{ij}(t),\ y_{ij}(0) = y_{ij0},\ i = 1, \ldots, N$

If there are some budget vacancies

- $J_0 = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{0j} x_{ij}^{B}(t) + const \rightarrow max;$
- $x_{ij}^{B}(t) \geq 0, a_{ij}^{B}(t) \geq 0$;
- $\sum_{i=1}^{N} \sum_{j=1}^{M} a_{ij}^{B} x_{ij}^{B}(t) \leq B$
- $\sum_{i=1}^{N} x_{ij}^{B}(t) + const \geq x_{j}^{min};$

a_{ij}^{B} - are minimal, equals prime cost of education

Reduce to the linear programming problem

- $J_0 = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{0j} x_{ij}^{B}(t) + const \rightarrow max;$
- $x_{ij}^{B}(t) \geq 0,$
- $\sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} x_{ij}^{B}(t) \leq B$
- $\sum_{i=1}^{N} x_{ij}^{B}(t) + const \geq x_j^{min};$

There is no condition of sustainable development

- $J_0 = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{0j} x_{ij}^{B}(t) + const \rightarrow max;$
- $x_{ij}^{B}(t) \geq 0,$
- $\sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} x_{ij}^{B}(t) \leq B$

Solution:

All budget vacancies should be allocated to the orientation with maximal significance (teachers).

All budget places should be allocated to a university that provides the minimal prime cost of education (DSTU)

There is a condition of sustainable development

- $J_0 = \sum_{t=1}^{T} \sum_{i=1}^{N} \sum_{j=1}^{M} a_{0j} x_{ij}^{B}(t) + const \rightarrow max;$
- $x_{ij}^{B}(t) \geq 0,$
- $\sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} x_{ij}^{B}(t) \leq B$
- $\sum_{i=1}^{N} x_{ij}^{B}(t) + const \geq x_j^{min};$

Solution.

To allocate a minimum number of budget vacancies for all orientations of the university, which provides the minimal prime cost of education (mainly DSTU).

To allocate all rest budget vacancies to the orientation that has the maximal public significance (teachers) to the university, which provides the minimal cost of education (DSTU)

Question

$\sum_{i=1}^{N} x_{ij}^{B}(t) + const \geq x_j^{min};$

- Who must define the values x_j^{min} if even MSE has no sense of it ?

The payoff of the center with the function with const proportions

- $J_0 = \sum_{t=1}^{T} \min_{1 \le j \le n} \{ \frac{\sum_{i=1}^{N} x_{ij}^{B}(t)}{a_{0j}} \} \rightarrow max;$
- $\sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} x_{ij}^{B}(t) \le B$
- $x_{ij}^{B}(t) \ge 0$;

The payoff of the center with the function with const proportions

- $J_0 = \sum_{t=1}^{T} \min_{1 \le j \le n} \{\frac{\sum_{i=1}^{N} x_{ij}^{B}(t)}{a_{0j}}\} \rightarrow max;$
- $\sum_{i=1}^{N} \sum_{j=1}^{M} c_{ij} x_{ij}^{B}(t) \le B$
- $x_{ij}^{B}(t) \ge 0$;
- zam= $\min_{1 \le j \le n} \{\frac{\sum_{i=1}^{N} x_{ij}^{B}(t)}{a_{0j}}\}$;
- $\sum_{i=1}^{N} x_{ij}^{B}(t) = zam \cdot a_{0j}$
- $x_{1j}^{B}(t) = zam \cdot \ a_{0j} - \sum_{i=2}^{N} x_{ij}^{B}(t)$

The payoff of the center with the function with const proportions

- $J_0 = zam \rightarrow max;$
- $\sum_{j=1}^{M} c_{1j}(zam \cdot \ a_{0j} - \ \sum_{i=2}^{N} x_{ij}^{B}(t)) + \sum_{2=1}^{N} \sum_{j=1}^{M} c_{ij} x_{ij}^{B}(t) \leq B$
- $x_{ij}^{B}(t) \geq 0, \mathrm{zam} \geq 0;$ i=2,…,n; j=1,…,m;

The payoff of the center with the function with const proportions

- $J_0 = zam \to max;$
- $zam \cdot \sum_{j=1}^{M} c_{1j}\ a_{0j} + \sum_{2=1}^{N} \sum_{j=1}^{M} (c_{ij} - c_{1j}) x_{ij}^{B}(t) \leq B$
- $x_{ij}^{B}(t) \geq 0, \mathrm{zam} \geq 0;$ i=2,…,n; j=1,…,m;

Solution

The number of budget seats is found non-trivially

For Rostov Region if the budget is 5 billion rubles the optimal allocation of budget vacancies is

Orientation	Teacher	Physician	Economist	Engineer	Builder	Agrarian
Budget vacancies	1988	1173	816	459	331	331

According to the results of modeling, all budget vacancies should be allocated to the universities that provide the minimal prime cost of educations (DSTU and RSMU)

Computations of demand for commercial vacancies

There is no demand for commercial vacancies in RSMU.

There is a demand for commercial places in DSTU, but the university does not offer commercial places. DSTU has not possibilities for educational large amount of budget students.

In SFU, the situation remains unchanged, the demand and possibility for commercial education remains.

Allocation of commercial vacancies

	RSMU		DSTU		SFU	
	Vacations	Extra charge	Vacations	Extra charge	Vacations	Extra charge
Physician			(0, 0, 0)	0%	(250, 250, 250)	11%
Teacher	(0, 0, 0)	0				
Engineer			(0, 0, 0)	0%	(250, 250, 250)	8%
Builder			(0, 0, 0)	0%	(86, 98, 9)	18%
Economist			(0, 0, 0)	0%	(28, 27, 5)	1%
Agrarian			(0, 0, 0)	0		

Conclusion and recommendations

It makes sense for the Regulatory Center (Ministry of Education and Science of the Russian Federation) to allocate all budget vacancies in the areas of higher education, which ensures the minimal prime cost of training (RSMU and DSTU)

If the university has been allocated all budget vacancies in the field of training, then there are no resources left for commercial places. The offer of extra-budgetary commercial vacancies in this case leads to losses (the model does not consider the possibility of the university's refusal of allocated budget places)

The economic feasibility of offering a large number of commercial vacancies is preserved only in the absence of a budget set (SFU)

The situation with the allocation of a small number of budget vacancies shows economic advantages, allowing the university to declare a maximum of commercial vacancies within the resource potential and, accordingly, receive more income.

Recommendations to universities. To determine the priority of the set: only budget or only commercial. To agree on specialization in the profiles of training specialists in the region without competition. SFU – teachers and economists, DSTU – engineers, builders and farmers, RSMU – doctors.