2nd International & European Conference
"Modelling and Simulation of Social-Behavioural Phenomena in Creative Societies"
(MSBC-2022)

FREE-RIDER PROBLEM: SIMULATING OF SYSTEM CONVERGENCE TO STABLE EQUILIBRIUM STATE BY MEANS OF FINITE MARKOV CHAIN MODELS

Olga E. Pyrkina, Andrey Yu. Yudanov

September 21-23, 2022
Vilnius, Lithuania

FREE-RIDER PROBLEM

- "Free-rider" activity is broadly
- defined in advanced economic
- theory as consumption of
- public goods without bearing the costs of its production in a share proportional to propensity to consume.

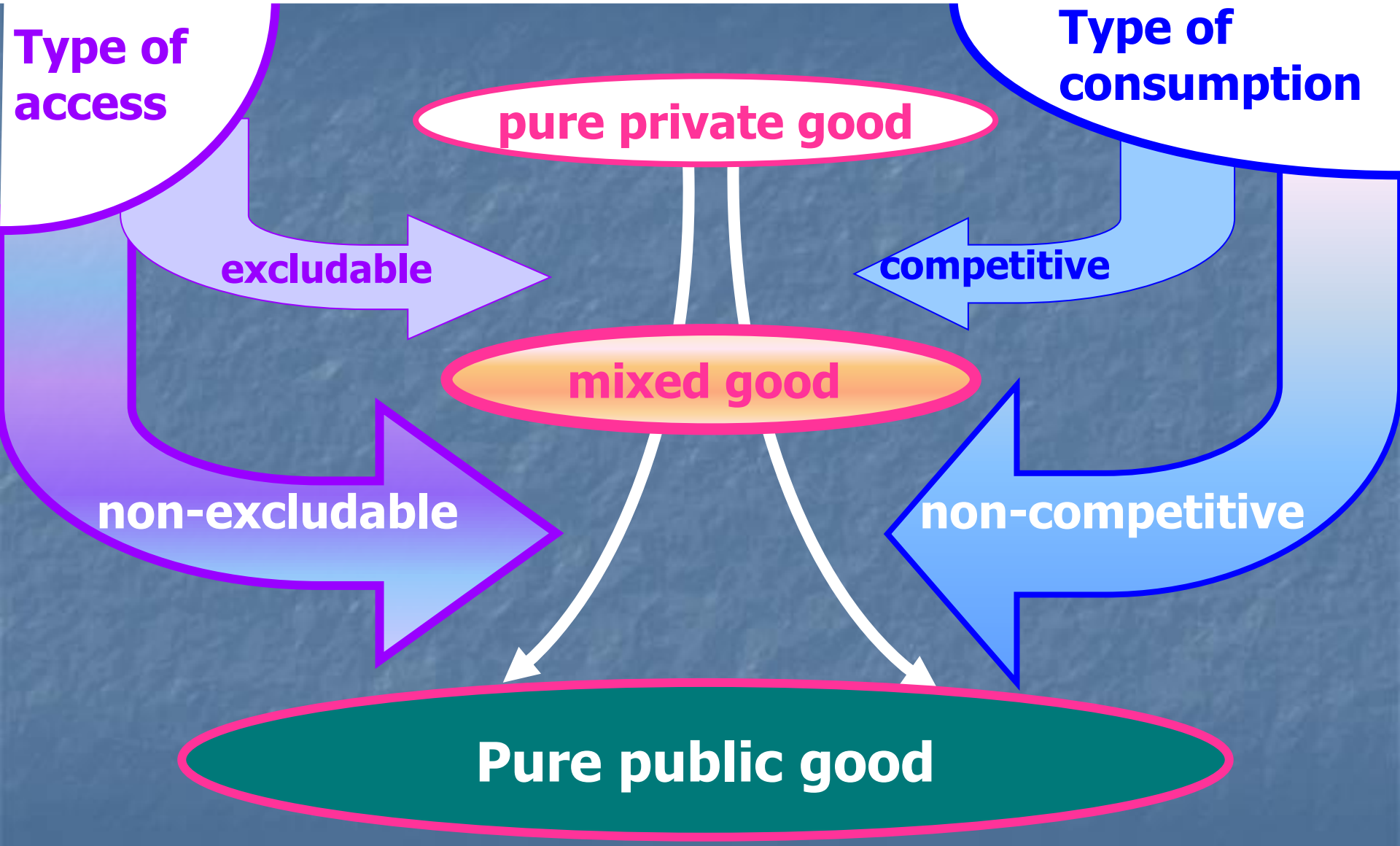
- A well-known in classic economic literature "free-rider" problem can be regarded to set of system failings of market economy.
  - The problem can't be solved by administrative measures, fines and other penalties.

fraudulent trick → access to benefits to which the free rider is not entitled

Widespread of the problem and attempts to solve it

- In economic problems
  - "shirking problem‟ in the theory of corporate governance.
  - "the problem of collective action‟ in public choice theory;
  - "resource sharing problem‟ in the theory of environmental management and ecology
- In the social sphere
  - Cheating on university exams
  - Movie piracy
  - counterfeiting

Economic classification of goods



Limits of non-competitiveness in consumption

SUBJECTS OF OPPOSITIONS TO "FREE-RIDING" AT DIFFERENT LEVELS OF PUBLIC AND PRIVATE DETRIMENT

	High private detriment	*Low private detriment*
High public detriment	**A**. Olson's small groups	**B**. State
Low public detriment	**C.** Immediate circle	**D.** Absent/State

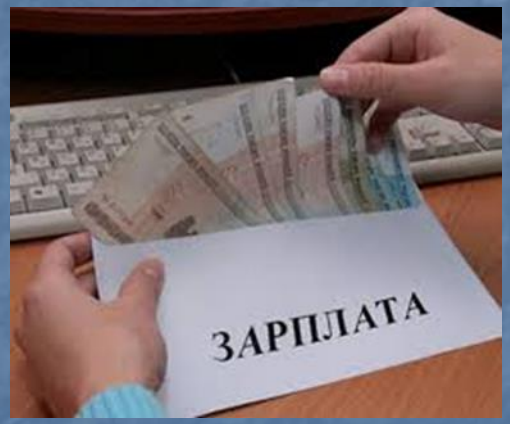
Intolerant position of society

Tolerant position of society

Situation with low total and high private damage

- Model
  - "counterfeiter problem"

- Model
  - "problem of off-the-book wages”



The Markov property.

For a Markov process, knowing the outcome of the last experiment we can neglect any other information we have about the past in predicting the future. It is important to realize that this is the case only if we know exactly 'the outcome of the last experiment. This condition says essentially that, given the present, the past and future are independent on each other.

John G. Kemeny, J. Laurie Snell: Finite Markov Chains. 1976, Springer -Verlag

A graph of possible movement for counterfeit banknote

$$F(1)=\begin{pmatrix} p_{11} & p_{12}d & p_{13}D & p_{12}(1-d)+p_{13}(1-D) \\ p_{21} & p_{22} & p_{23}D & p_{23}(1-D) \\ p_{31} & p_{32} & p_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$d = e^{-\gamma q}$$

$$D = e^{-\Gamma q}$$

$$\vec{x}(0) = \{1, 0, 0, 0\}$$

$$\vec{x}(n) = \vec{x}(0) P^n(1)$$

$$\vec{x}(t*) = \{0, 0, 0, 1\}$$

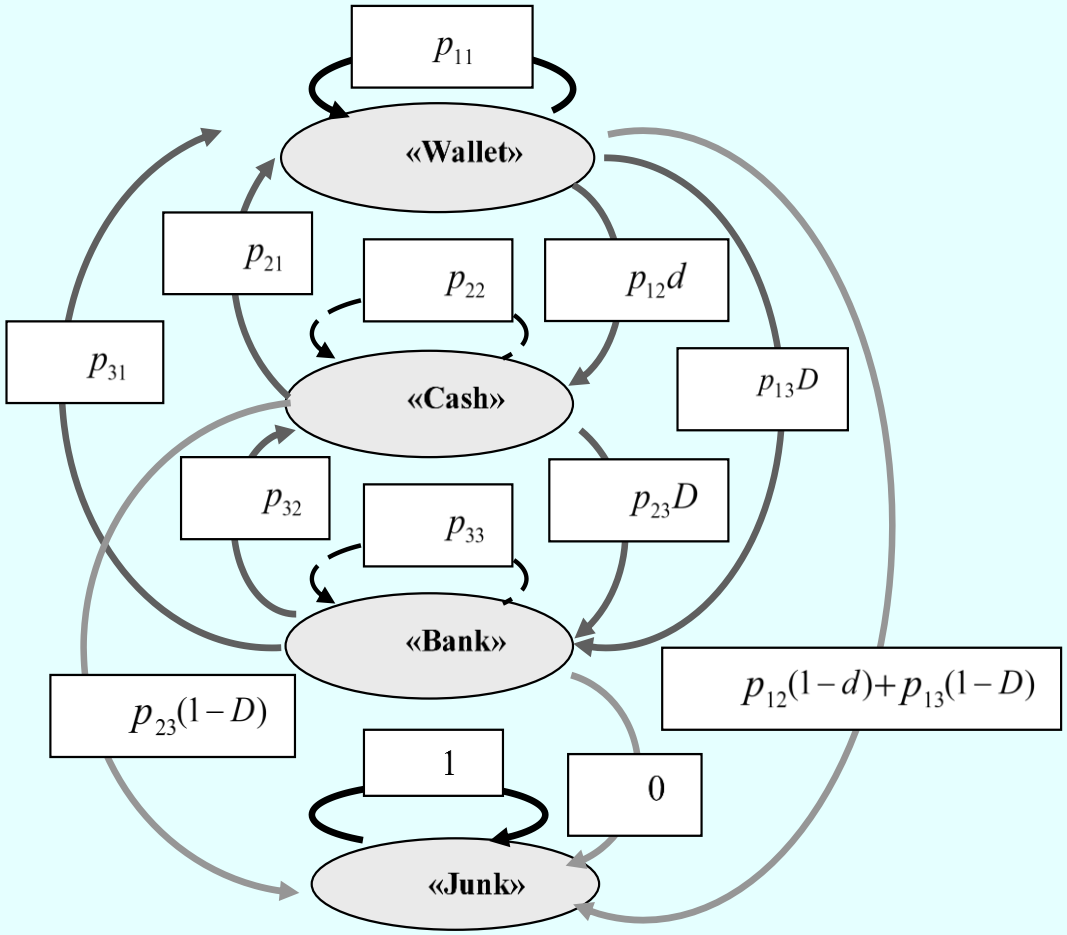


Fig. 1. The graph of possible movements of counterfeit banknote

Dependence of a number n of time steps before the system achieves an absorbing state on the concentration q of counterfeit banknotes in circulation

Simulation Example $\gamma = 1, \Gamma = 2$

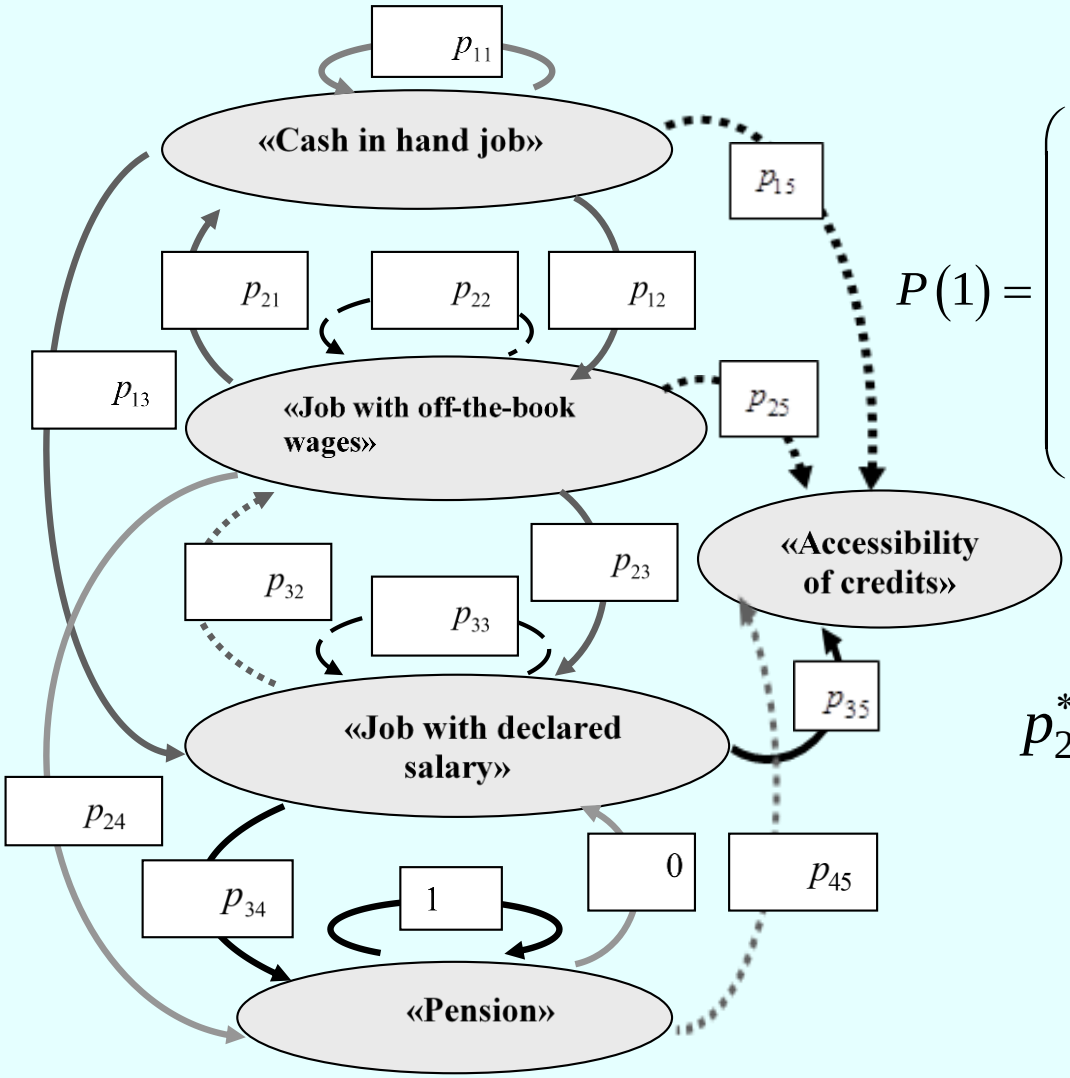
$$P(1) = \begin{pmatrix} 0.4 & 0.4 & 0.2 \\ 0.4 & 0.2 & 0.4 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$$

The absorbing state is reached if

$$X = \{1, 0, 0, 0\} \longleftrightarrow X = \{\leq 0.01, \leq 0.01, \leq 0.01, \geq 0.99\}$$

$q = 0.02$	$q = 0.05$	$q = 0.1$	$q = 0.2$	$q = 0.3$
n>300	n=143	n=83	n=40	n=29

Graph of employee relocation to different states of employment



$$P(1) = \begin{pmatrix} p_{11} & p_{12} & p_{13} & p_{14} & 0 \\ p^*_{21} & ce^{-\alpha D} & c\left(1-e^{-\alpha D}\right) & p^*_{24} & 0 \\ p_{31} & p_{32} & p_{33} & p_{34} & p_{35} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$p^*_{21} + ce^{-\alpha D} + c\left(1-e^{-\alpha D}\right) + p^*_{24} = 1$$

$$D = \frac{\text{market "declared" salary for position} - \text{"declared" part of "off-the-book" salary}}{\text{"declared" part of "off-the-book" salary"}}$$

Dependence of a number n of time steps before the system achieves an absorbing state on the value D of governing parameter

Simulation Example

$$c = 0,5$$

The absorbing state is reached if

$$X = \{0.2, 0.2, 0.2, 0.2, 0.2\}$$

$$\Longleftrightarrow \quad X = \{\leq 0.01, \leq 0.01, \leq 0.01, \geq a, \geq b\}, a + b = 0.99$$

$D = 2$	$D = 5$	$D = 10$	$D = 20$	$D = 30$
n>300	n=120	n=72	n=17	n=6

Public opinion and cheating

Opinion \ country	Russia	USA
Cheating person	–0.24	–1.35
Giver cheat	0.52	–0.88
dime-dropper	–1.78	–0.25

Jan R. Magnus, Victor M. Polterovich, Dmitri L. Danilov, and Alexei V. Savvateev
Tolerance of Cheating: An Analysis Across Countries
JOURNAL OF ECONOMIC EDUCATION, Spring 2002, p. 127

CUI PRODEST?
WHOSE PROFITS?

- "look for those for whom it is UNprofitable".

Thank you for attention!

yudanov@yandex.ru

olga.pyrkina@gmail.com